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By T. Theodorsen
Langley Memorial Aeronautical Laboratory

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## SUMMARY

A method has been devised for determining the frequencies of the various modes of a stationary propeller and the associated crankshaft. A method has also been devised to obtain the effect of the centrifugal force on a revolving propeller by the use of a flexible model.

## INTRODUCTION

A noticeable increase in the number of propeller failures has recently focused the attention rather strongly on this problem. The status of the knowledge on this question will be briefly indicated in this paper. Some of the recent work by the National Advisory Committee for Aeronautics is presented and an attempt made to give certain tentative recommendations with respect to future practice, insofar as this is definitely possible. Propeller failures may apparently be subdivided into two classes: shank failures and tip failures. The various modes of vibration that a propeller is capable of performing are first described and convenient methods of determining them explained.

In a first approximation, the propeller is a tapered beam symmetrical with respect to the hub axis. A beam supported in the middle is capable of performing two types of vibrations, which shall be referred to as "symmetrical" and "nonsymmetrical". The three lowest modes of each type are indicated schematically in figure 1.

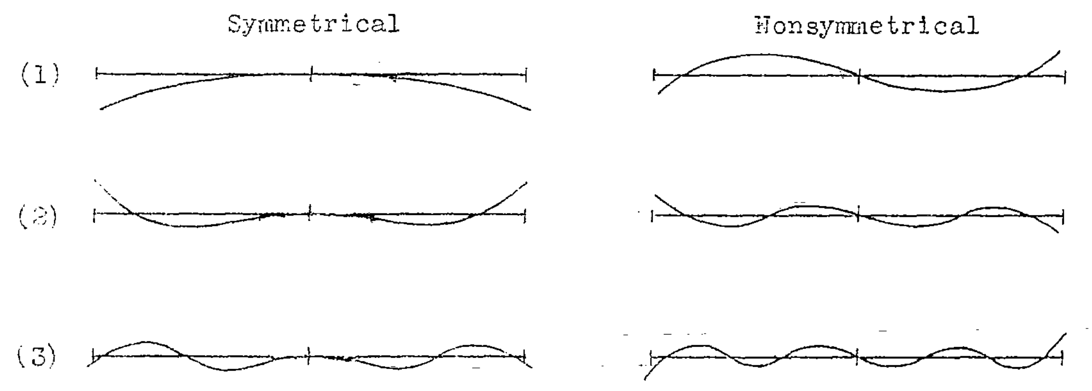


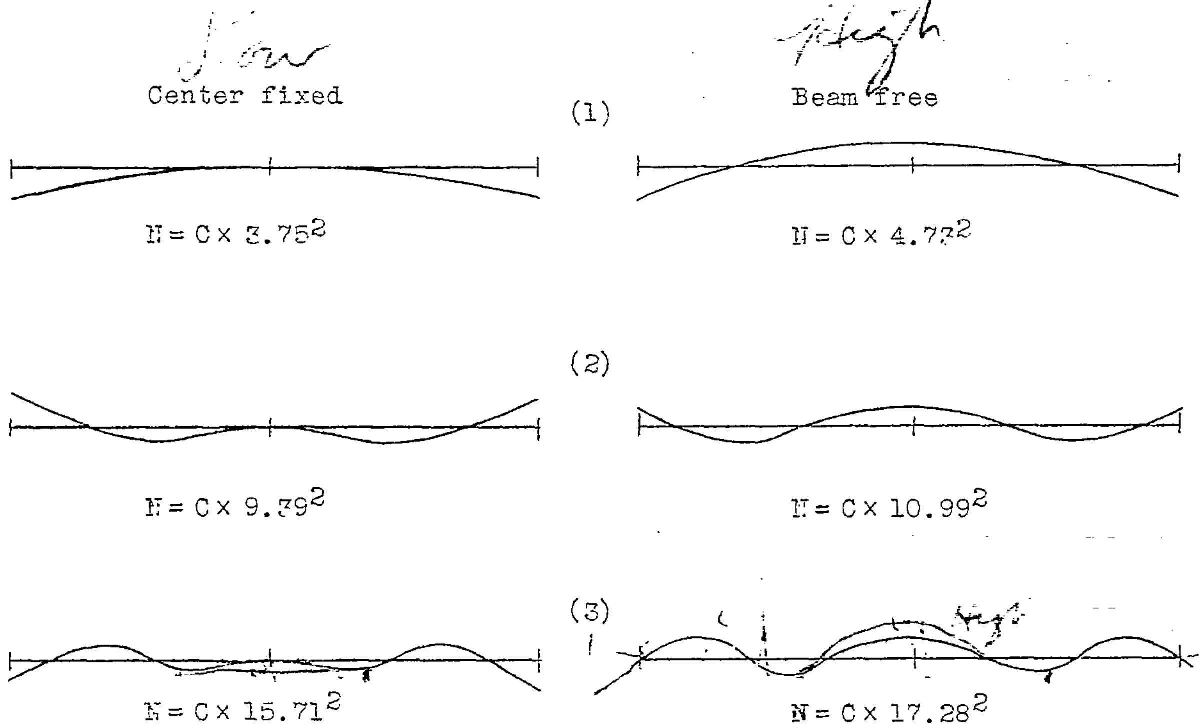

Figure 1.

In regard to the symmetrical type of vibrations it should be noticed (fig. 1) that the point of symmetry is fixed or prevented from performing a perpendicular motion. This condition is strictly in accordance with the situation of a propeller mounted on the engine. The fore-and-aft motion of the propeller hub is of the order of some thousandths of an inch, while the motion of the tip in any case of concern is several hundred times larger. With the center fixed or free, the difference in the frequency of vibration is quite considerable in the case of a rectangular beam, for which the frequencies are known. (See reference 1.)

Figure 2 gives a comparison of the two cases, including the frequency formulas. The free beam is therefore seen to execute frequencies that are higher than those of the beam fixed at the center by the following factors:

| | |
|---|---|
| First mode: | 1.61 |
| Second mode: | 1.38 |
| Third mode: | 1.21 |

The results are illustrative, but of course almost misleading as to magnitude, as the effect is much less pronounced on a tapered beam or a propeller, in particular with respect to the third mode, which is ordinarily of most concern. The following discussion will be restricted to the type with the center fixed, as being the only one of interest.




$$C = \frac{K}{2\pi L^2}\sqrt{\frac{E}{\rho}}$$

L = length of beam.
K = the radius of gyration

Figure 2

In order to create this type experimentally, it is very convenient to mount the propeller on a shaft and then subject the shaft to an alternating torsional twisting moment. To obtain the twisting moment, the National Bureau of Standards has employed an electric motor mounted on the shaft and supplied with an alternating current of the desired frequency (reference 2). The N.A.C.A. has adopted a slightly different scheme, convenient because of its simplicity and the possibility of obtaining as many of the higher modes as are desired, usually as high as the fifth order.

## MEASUREMENT OF PROPELLER FREQUENCIES

This method consists in mounting the propeller directly on the engine to which it belongs and mounting a high-frequency vibrator on the crankpin. (See figs. 3 and 4.) The vibrator is a small air-driven turbine with a slight unbalance of the mass. The type used at present is capable of running up to 30,000 r.p.m. The unbalance can be altered to suit the requirement. (See fig. 5.) The vibrator turbine is connected to a regular precision tachometer through a 20:1 reduction gear. It is mounted on the crankpin with the axis parallel to the engine shaft. The unbalance is adjusted to give a measurable deflection. The frequency of the various modes are then directly read on the tachometer.

For those not entirely familiar with the subject, it should only be mentioned that the lowest mode is of the order of 2,000 cycles per minute; the second around 6,000; and the third approximately 10,000 to 12,000 cycles. The engine is not capable of producing vibrations other than of the symmetrical type. They are further restricted to the fixed-center class. The former statement is obvious, since the impulses from the engine act on both or all propeller blades in identical manner. The reason for the second statement has already been given.

The vibration frequencies have been obtained at the Langley Memorial Aeronautical Laboratory on a number of conventional propellers; but, since the experiment can very easily be reproduced, these are of little general interest. As is already known, the angle of pitch has a noticeable influence on the frequency. The effect is not very large, however, through the ordinary range of pitch angles.

Regarding the location of the various nodes, these may readily be obtained either by the dust method commonly used or by a vibration-amplitude recorder of the type described in reference 3, which method is used by the Laboratory. It should be emphasized, however, that an exact determination of these nodal points serves no particular purpose since, first, they are considerably altered by the effect of the centrifugal force and second, because a propeller is not expected to be operated in a state of critical vibration, as shall be pointed out more clearly. Only as a matter of explaining or analyzing definite cases or propeller failures are these nodal points of interest, as they help to establish, to a certain degree, the exact location of expected maximum strains. In fact, the interest in the strain due to vibration is only for the purpose of explaining why certain failures occurred; thus learning how to avoid certain dangerous conditions.

The following paragraphs will be devoted to a discussion of how to alter the shape of the propeller so as to obtain more or less specified vibration frequencies; or, in other words, to point out how any one of the various modes may be made to appear at higher or lower frequencies.

The frequency of a certain mode of vibration is given by the formula

$$N = \frac{D}{2\pi L^2}\sqrt{\frac{E}{\delta}}\, f(s) \quad \text{(See reference 1, page 361).}$$

where $L$ is the length of the propeller blade and $D$ is a quantity representative of the thickness, say the diameter at the hub; $f(s)$ is a function of the relative shape of the propeller and depends also on the mode. The number $f(s)$ is the same for all similar blades; that is, similarity in cross section for similar locations along the length $L$. Starting with a certain original propeller, all dimensions along the length axis may be increased in a certain proportion, or all dimensions in the plane perpendicular to the length axis be increased in a certain ratio, or both. The function $f(s)$ will not be altered by this process. Note that this kind of similarity differs from the ordinarily used definition in the fact that two factors of proportionality are involved, one giving the ratios of cross sections and the other the ratio of lengths. The resulting "similar" bodies are all perfectly orthodox propellers, and furthermore it can be shown that all types of propellers are rather closely related so that the function $f(s)$ is not very appreciably altered.

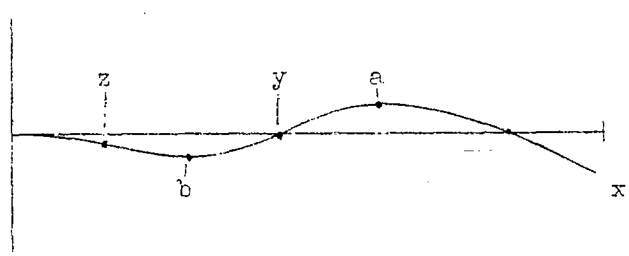
The first important rule is: The frequency of the various modes of vibration varies directly with the thickness and inversely with the square of the length.

If an increase in the frequency of any particular mode is desired (a problem frequently encountered), and this result is not wanted at the expense of an increase in weight of the propeller or an alteration in its principal dimensions, the procedure of reasoning is as follows:

The frequency of a certain mode is dependent on the ratio of the energy stored in the deflected structure in the particular mode to a certain effective mass of the structure. Near the nodal points, stored energy is small since the curvature is small. Near the loops, the stored energy is considerable and is proportional to the square of the curvature (or deflection), and to the quantity EJ. The frequency may be increased by increasing the stored, or potential, energy in the region near the greatest curvature. A second rule can therefore be expressed very simply:

To increase the frequency of vibration of a certain mode increase the stiffness at points near greatest curvature (loops) by removing mass from points near small curvature.

In order to illustrate the method, refer to figure 6 showing the third mode of vibration (symmetrical, center fixed).




Axis Figure 6.

Note that points of greatest curvature are a, b, and some point c near the hub; the points of least curvature are the tip x, some point near the middle node at y, and the inner point z. Because of the small deflections near the hub, the central portion has a very small effect on the frequency. The procedure in raising the frequency of the third harmonic consists primarily in removing mass from the tip to the region near a in the middle of the outer loop, and secondarily by removing mass from the point y near the middle node to the point b in the middle of the inner loop. It should be noted in addition that the thickness-chord ratio at a and b should be as large as permissible, since the object simply is to increase the moment of inertia around a line in direction of the chord.

The nodes measured on a 9-foot propeller at the Laboratory for the third mode are given as an illustration in figure 7.

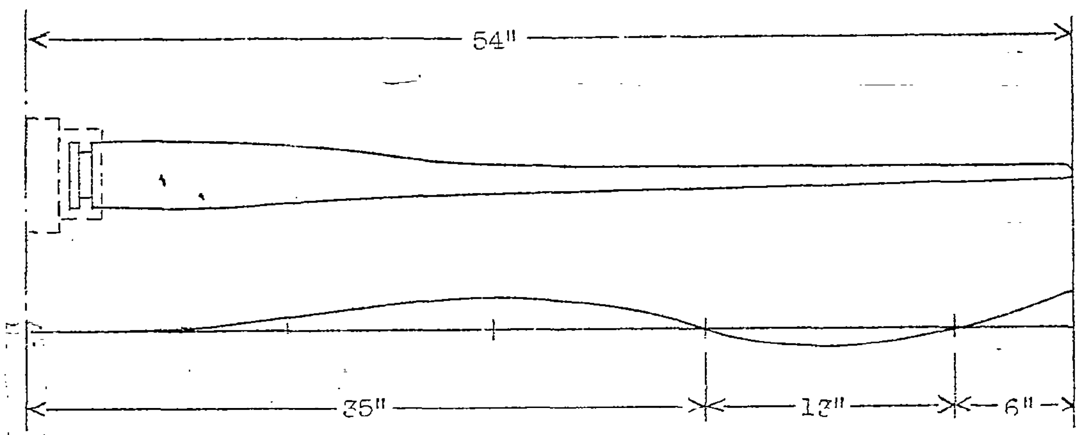



Figure 7.

It is evident from the figure that, in order to raise the frequency of the third-order vibration, the particular propeller shown should be thickened at the stations: approximately 41 inches, 25 inches, and hub. In order to save weight it should be thinned out at the stations: tip (54), 35, and approximately 15.

The identical or reverse procedure may be applied to lower or raise any one of the various frequencies.

## THE EFFECT OF CENTRIFUGAL FORCE ON PROPELLER VIBRATIONS

Elementary considerations show that the centrifugal force acting on a revolving propeller increases the vibration frequencies of the various harmonics. The question then is how far the frequencies obtained on a nonrevolving propeller may be used to consider actual conditions. It shall be shown that the effect is considerable. It is subject to a rather straightforward computation, which, however, in the case of a true propeller shape becomes very laborious. The method is given by Hohenemser (reference 4), who carries through the calculation for the three lowest modes on a simplified (triangle shaped) length section. The author has devised a very convenient method for determining the effect. It is based on the following reasoning.

The frequency in any mode depends on the ratio of stored potential energy of the deflected structure to the mass of the structure times the square of the deflection. The stored potential energies are now two kinds, viz, the bending energy $E_b$ and the energy due to the centrifugal force $E_c$. If the ratios of the two kinds of stored energies are kept constant, there results a frequency of vibrational proportional to the original pure bending frequency.

The frequency due to bending has been shown to be given by

$$f_b = \frac{D}{L^2} V_s \, f(s)$$

where $V_s = \sqrt{\frac{E}{\rho}}$ (sound velocity in the material). The frequency due to centrifugal force alone is given by

$$f_c = w \, f_1(s) \quad \text{(see reference 1, page 367)},$$

where $w$ is the frequency of revolution and $f(s)$, like $f_1(s)$, depends only on the shape and is definite number for the defined similar propellers.

The resulting frequency in any particular mode $y = f(x)$ is given by

$$f^2 = f_b{}^2 + f_c{}^2$$

By the simple expedient of keeping the ratio of the expressions $f_b$ to $f_c$ constant, a mathematically correct reproduction of the relative frequencies is obtained. This ratio is for similar propellers, except for a constant

$$R = \frac{D}{L^2} \frac{V_s}{w} = \frac{D}{L} \frac{V_s}{V_T}$$

where $V_T$ is the tip velocity.

The rule is obtained:

The relative increase in frequency due to centrifugal force in <u>similar</u> propellers is the same for the same value of the quantity $R = \frac{D}{L} \frac{V_S}{V_T}$.

This fact is very useful. It is obviously very difficult to obtain experimental results on a full-size propeller. Identical increases due to the centrifugal force are obtained by reducing the cross section, however, say 10 times, and running the resulting thin propeller at one-tenth speed. For a full-scale propeller showing a frequency of 2,000 cycles per minute at rest and, say 3,000 cycles per minute at 2,000 r.p.m., there is obtained on the thin reproduction 200 cycles per minute at rest and 300 cycles per minute at 200 r.p.m.

The experiment is very easy to perform on this thin copy. The horsepower requirement is down to one thousandth of the former value. It was found desirable to enclose the entire propeller in a protecting tube to prevent the air damping from influencing the results.

The experimental installation is shown in figure 8 with the tube off and in figure 9 with the tube on. The propeller is mounted horizontally to avoid any effects of gravity. A vibrator is mounted near the hub to impose vibrations at any desired frequency in a vertical direction. Critical frequencies make themselves evident by the propeller tip hitting the metal tube, which acts as a bell. The propeller is run at a certain revolution speed and the vibrator is gradually speeded up until the sound from the tube is heard. The maximum is carefully adjusted and noted.

The results of the first experiment of this type are given in figure 10. The abscissa is here the revolution speed of the propeller and the ordinate is the vibration frequency. (Both values have been multiplied by 10.) Although the results may be considered somewhat preliminary they are in remarkably good agreement with those by Liebers (references 5 and 6) for the fundamental and with the values calculated by Hohenemser (reference 4) for the second and third harmonics. Hohenemser gives for the value $c$ in the usual formula $f = \sqrt{f_0^2 + c\,N^2}$ the values $2.5 < c < 3.9$ for the second and $c < 12.2$ for the third, both calculated for the simplified case of the triangular length section perpendicular to the blade and constant width. The Laboratory experiment gives, for comparison, the values indicated in parentheses in the figure and reproduced in the following:

Values of c

| | N.A.C.A. measured experimentally | Hohenemser calculated for triangular length section |
|---|---|---|
| 1st mode | 1.7 | -- |
| 2d mode | 2.8 - 3.5 | 2.5 - 3.9 |
| 3d mode | 12 - 12.3 | $< 12.2$ |
| 4th mode | 21 - 24 | -- |

On the basis of the experience gained, it is the intention to refine the method to obtain greater accuracy. In particular the responses were not quite as definite as they could have been, owing to a damping effect of the rather heavy auxiliary supporting structure. A new design is under construction. It is planned to study the effect of particular shapes in more detail.

## THE EFFECT OF UNDESIRABLE PROPELLER CRANKSHAFT COMBINATIONS

Figures 11 and 12 show typical cases, each with two different propellers. The curved lines are the successive propeller modes, reconstructed by means of the experimental values obtained for c, and the horizontal line gives the measured crankshaft critical.

The latter is obtained in the same type of experiment as already described for the determination of the stationary propeller frequencies. Equivalent weights are used in the crankpins to replace the piston system. The auxiliary drives have also been removed. This procedure results in a very sharp and well-defined shaft critical. This value was found to be 9,800 cycles per minute for the P. & W. Wasp, corresponding to 2,180 r.p.m. for the 9-cylinder engine, and at 12,600 cycles/min. ~~r.p.m.~~ or 1,800 r.p.m. for the 14-cylinder 2-row R-1830 engine. The former result is in very good agreement with results obtained at Wright Field with the Prescott Indicator (reference 7, page 3). The second result appears to be 100 - 200 cycles per minute higher than similar results obtained by the Prescott Indicator. The present experiment was performed mainly to demonstrate the method, and no attempt will be made to explain minor discrepancies.

The conclusions to be drawn from the propeller-engine characteristics shown in figures 11 and 12 are that the former represents a desirable combination in the range 1,500 - 2,180 r.p.m. for one propeller and 1,300 - 2,180 for the second. The range could be extended downward, however, by employing a more flexible propeller than either of the two types shown. It is seen that the second mode goes into resonances with the explosion impulses of the engine at 1,500 and 1,300 for the two propellers, respectively, and that the third mode even for the second propeller (dotted) is beyond the range entirely. Conclusions from the second diagram (fig. 12) are that this is a very undesirable combination. At 1,800 r.p.m., not only the crankshaft critical but also the third mode of both propellers are in resonance with the explosion frequency of the 14-cylinder engine. Remedy: - Employ a propeller with the third mode at about 12,500 cycles per minute, and lower the shaft critical speed (if necessary by means of spring hub) to about 8,500 cycles per minute. This procedure will leave the range 1,250 to 2,000 r.p.m. clear. Diagrams of this type are considered indispensable in connection with aircraft-engine propeller installations.

## PROPELLER FAILURES

When the explosion frequency is in resonance with the shaft critical speed, there results a variable torque ~~of~~ considerable magnitude. From considerations of the moments of inertia and the known frequencies it is found that the twist of the crankshaft due to the full-load torque of the engine is of the order of $1/4^\circ$ for the radial types investigated. Let it be assumed that the variable torque amplitude is about one quarter of this average value, or about $1/16^\circ$. Now deflections have been observed in operation of $1^{\circ}$ and even as much as $2^{\circ}$, which means that the variable torque due to resonance has been increased by a factor of 16 to 32. This torque enters the propeller shank as a bending moment and, if the values given are fairly representative, the shank is subjected to an alternating bending moment of from 16 to 32 times the static value, or from 4 to 8 times the normal average bending moment resulting from the air load. It is obvious that the condition may be dangerous, insofar as the endurance limit may be exceeded. If the shaft is not running at its critical speed but in resonance with one of the propeller modes the strain is at normal value near the hub; whereas a tip breakage may occur.

The condition of double resonance, as was indicated to occur in the case reproduced in figure 12, is still more dangerous as far as a tip failure is concerned, although it will depend on the relative circumstances whether a shank or a tip failure will result.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., January 18, 1935.

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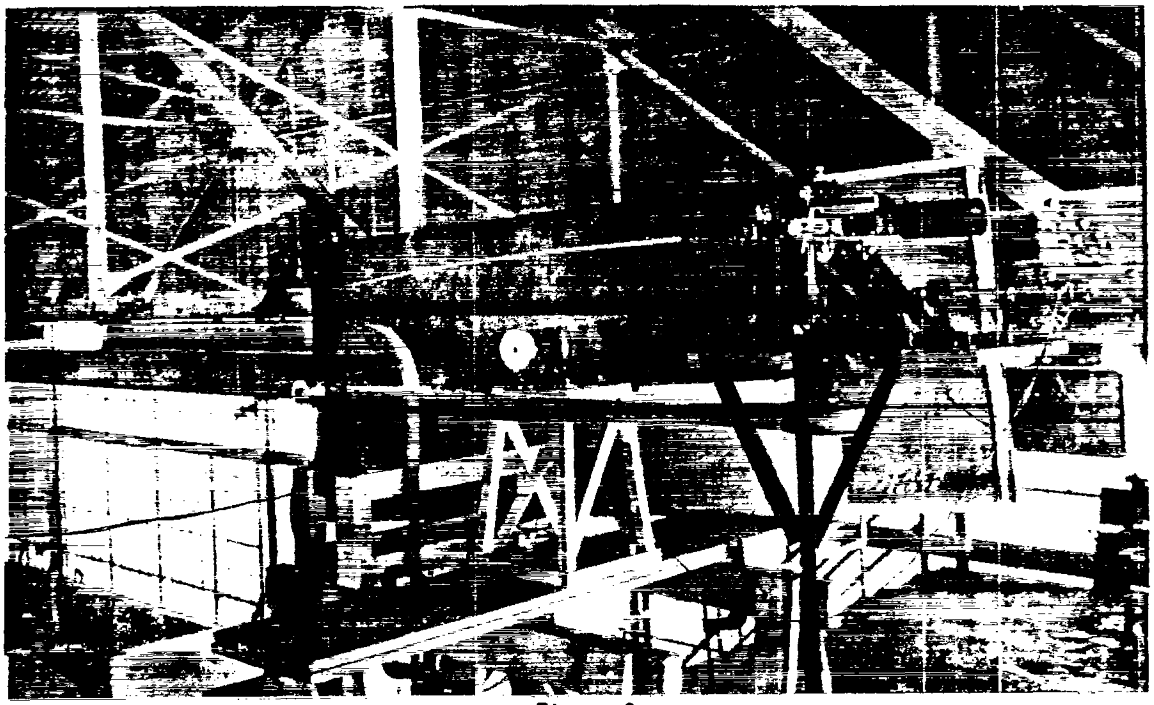

Figure 8.

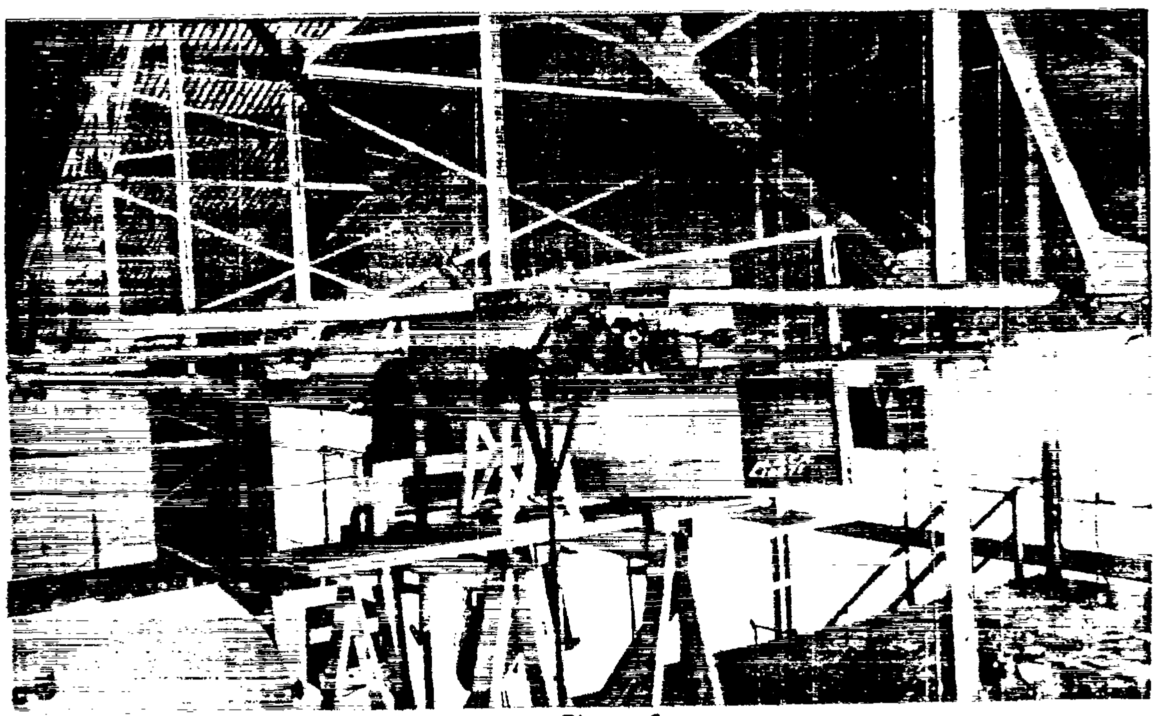

Figure 9.

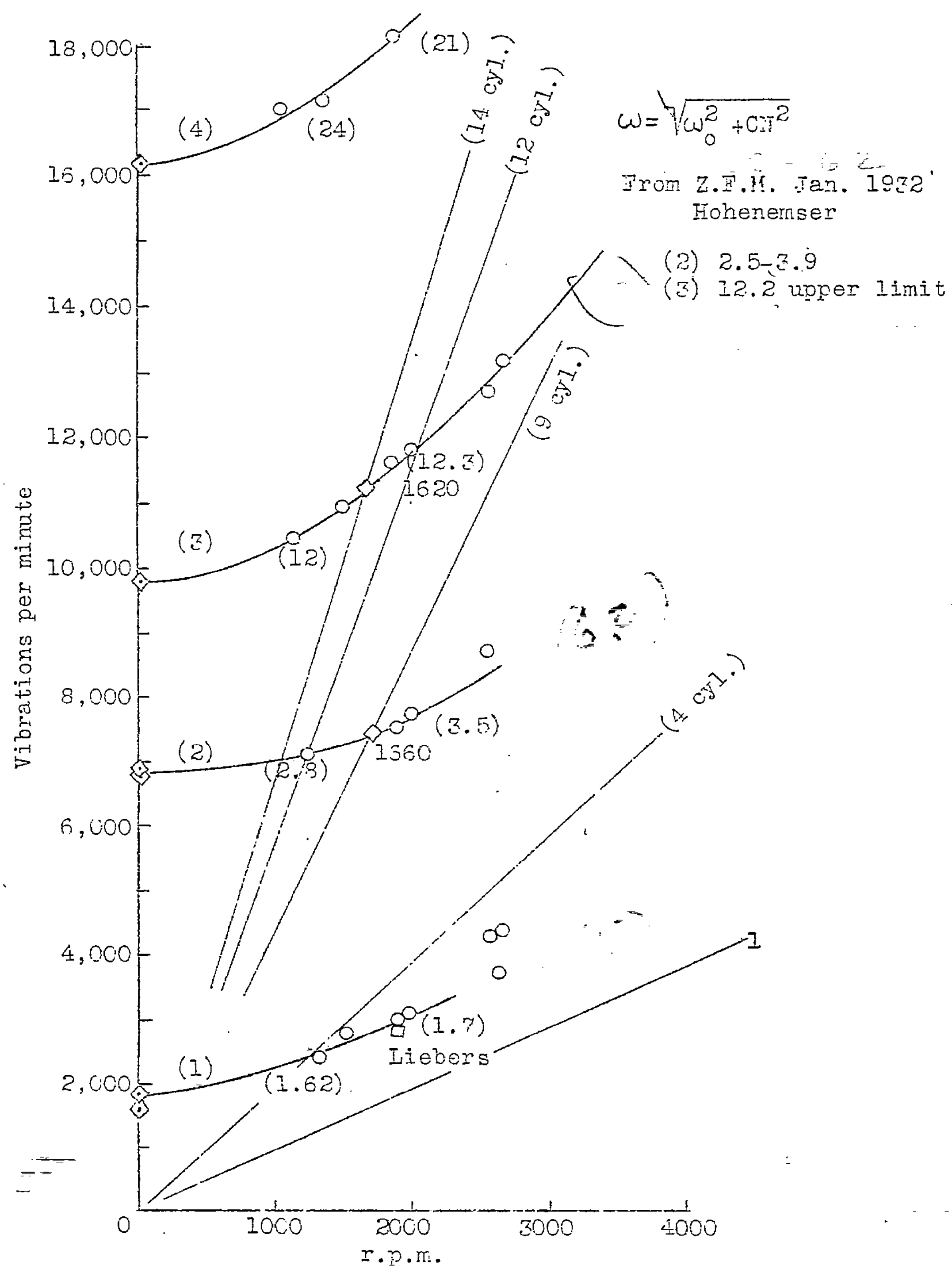


Figure 10.-Effect of centrifugal force on frequencies of propeller vibrations.

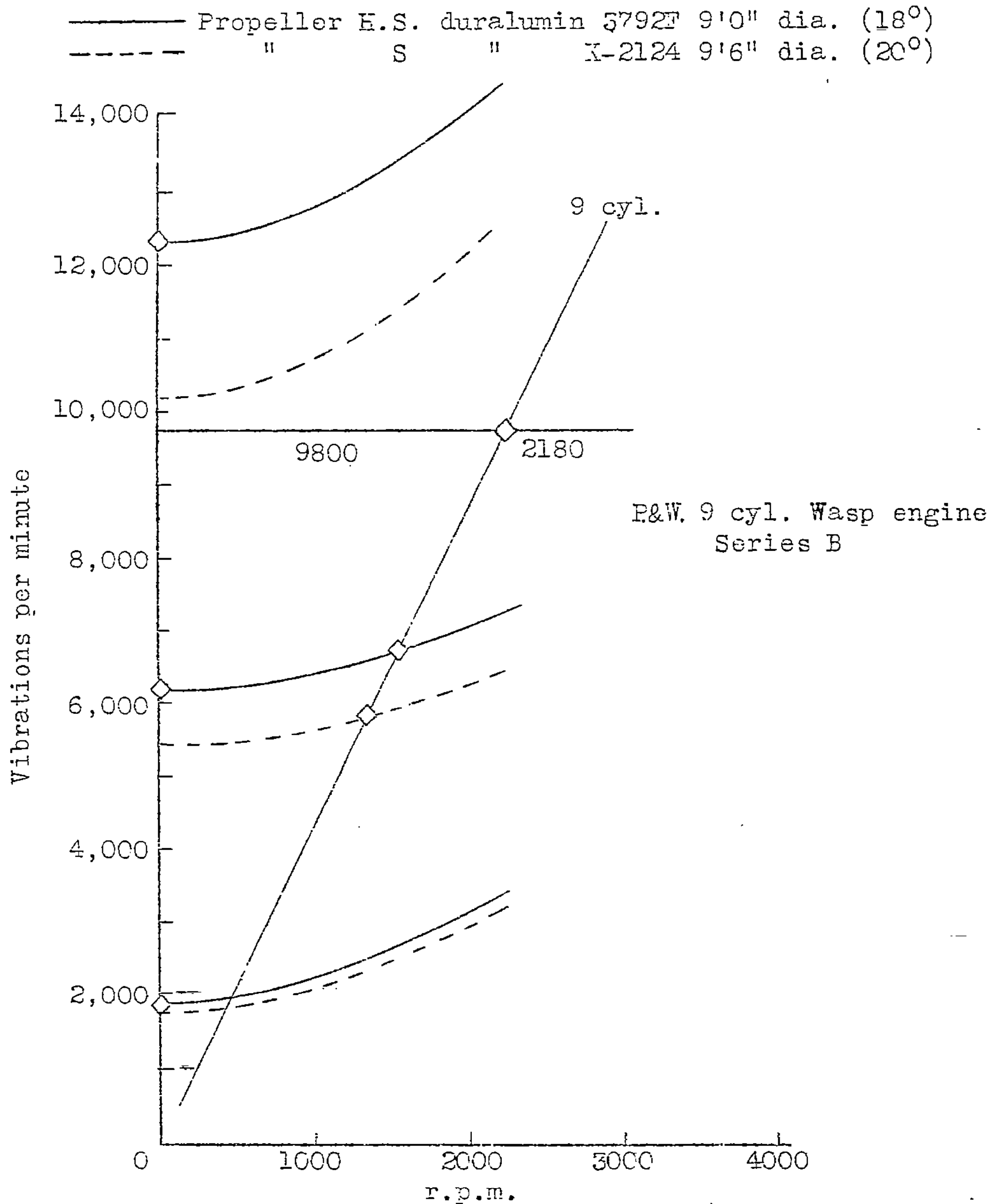


Figure 11.-Propeller crankshaft vibrations.

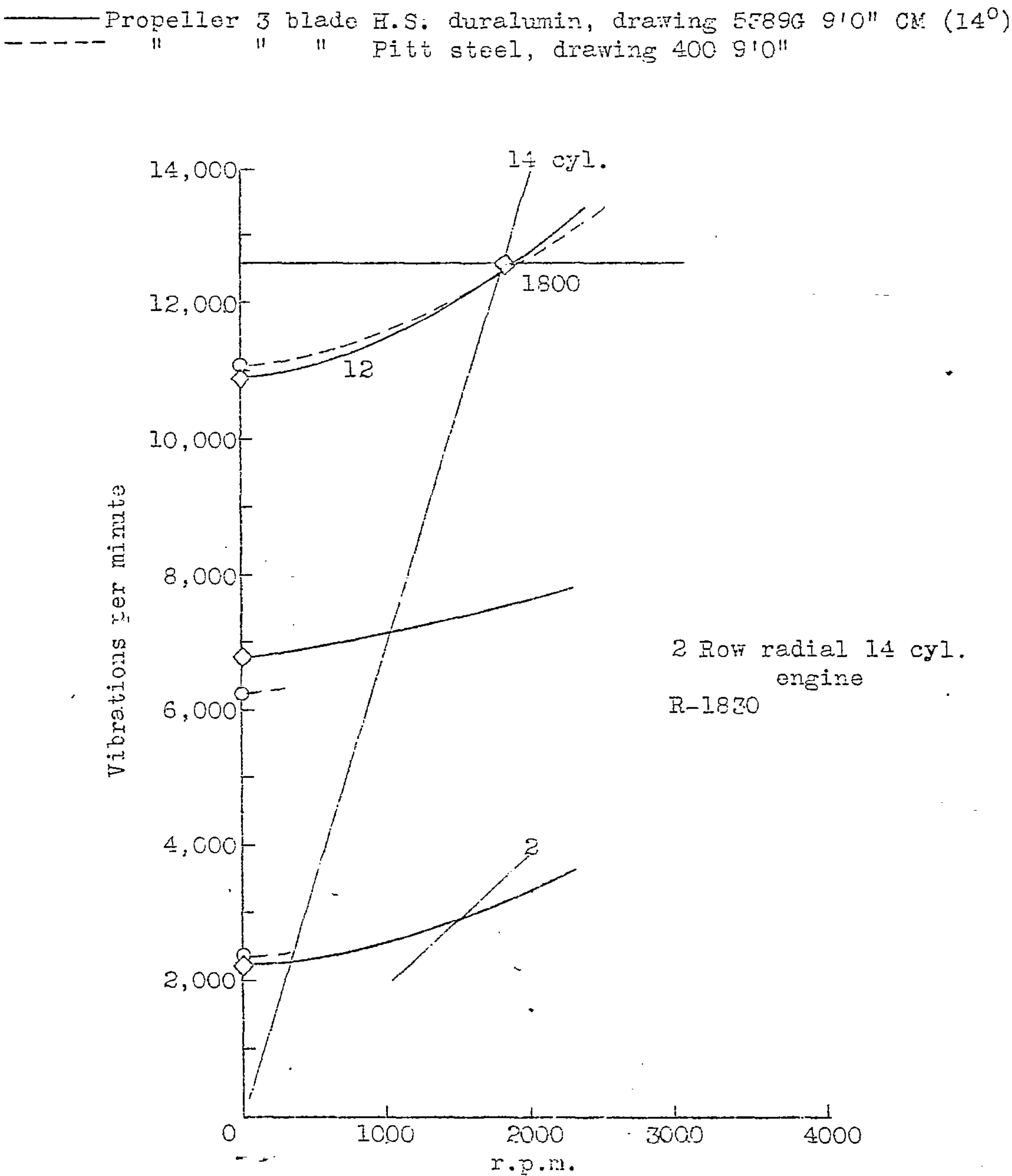


Figure 12-Propeller crankshaft vibrations.